

Reflection symmetry breaking scenarios with minimal gauge form coupling in brane world cosmology

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Abstract

This article synthesises and extends recent work on the cosmological consequences of dropping the usual Z_2 reflection symmetry postulate in brane world scenarios. It is observed that for a cosmological model of homogeneous isotropic type, the relevant generalised Birkhoff theorem establishing staticity of the external vacuum in the maximally symmetric “bulk” outside a freely moving world brane will remain valid for the case of motion that is forced by minimal (generalised Wess Zumino type) coupling to an external antisymmetric gauge field provided its kinetic action contribution has the usual homogeneous quadratic form. This means that the geometry on each side of the brane worldsheet will still be of the generalised Schwarzschild anti-de Sitter type. The usual first integrated Friedmann equation for the Hubble expansion rate can thereby be straightforwardly generalised by inclusion of new terms involving 2 extra parameters respectively measuring the strength of the gauge coupling and the degree of deviation from reflection symmetry. Some conceivable phenomenological implications are briefly outlined, and corresponding limitations are derived for possible values of relevant parameters.

1 Introduction of minimal coupling

The proverbially fundamental complementarity of “brain versus brawn” has an analogue in the purely mathematical context associated with names such as Cartan and de Rham wherein gauge equivalence classes of p -forms (i.e. covariant antisymmetric tensor fields) arise as the homological complement of p -surfaces. Conversely, in the higher dimensional physical context associated with names such as Polchinski and Witten, $(p - 1)$ -branes (i.e. p -surface supported systems) emerge naturally as concentrated source distributions for gauge p -form fields. Thus given any $(p - 1)$ -brane one of the first questions that comes to mind is what about the corresponding “brawn”, i.e. the associated gauge p -form? In the case of a point particle (i.e. a zero brane) the answer is usually that it is the ordinary Maxwellian electromagnetic field. In the case of the recently much studied theories that treat our world as a 3-brane in a 5-dimensional “bulk”, the question of the corresponding gauge 4-form has been generally overlooked, perhaps because the ensuing “brawn” force would have to vanish anyway in the reflection symmetric scenarios that are most commonly [1, 2] considered.

Several authors have discussed models in which the usual Z_2 symmetry postulate in homogeneous isotropic cosmological scenarios treating our four dimensional world as a 3-brane in a

5-dimensional “bulk” was dropped. Ida [3] and then Krauss [4] showed that such a reflection symmetry breaking can arise due to the choice of different integration constants for the solution of the vacuum Einstein equations on either side of the brane, and the general question of the observable consequences of dropping this symmetry been addressed by Davis *et al.* [5]. A different kind of symmetry breaking has been considered by Deruelle and Doležel [6] and Perkins [7], who considered bubble walls separating two distinct five dimensional anti-de Sitter spacetime domains with different cosmological constants. Finally, the possibility that both kinds of symmetry breaking occur simultaneously has been considered by Stoica *et al.* [8] and by Bowcock *et al.* [9], who have shown how to obtain a generalisation of the usual first integrated Friedmann equation giving the square of the Hubble expansion rate as a sum of terms in which, as well as the the ones that are now familiar [10] in “conventional” (meaning force free reflection invariant) brane world cosmology there is a new term (including the extra contribution considered in [3, 4, 5, 6, 7, 8]) that could have been important at some stages in the early universe.

The present article provides a synthesis that develops the underlying physics giving rise to such scenarios by incorporating allowance for the simplest conceivable coupling to an external antisymmetric “brawn” gauge field, $A_{\nu\rho\sigma\tau}$ say (where Greek indices run from 0 to 4). This minimal coupling case [11] is governed by an action consisting just of an external kinetic contribution of the usual purely quadratic type, together with a coupling contribution given by an action density of the (generalised Wess Zumino) form

$$\mathcal{L}_{\text{co}} = (4!)^{-1} \mathbf{j}^{\nu\rho\sigma\tau} A_{\nu\rho\sigma\tau} , \quad (1)$$

for a Dirac distributional source given by some dimensionless coupling constant $e_{\{4\}}$ as

$$\mathbf{j}^{\nu\rho\sigma\tau} = \overline{\mathbf{j}}^{\nu\rho\sigma\tau} \delta[\zeta] \quad \text{with} \quad \overline{\mathbf{j}}^{\nu\rho\sigma\tau} = e_{\{4\}} \mathcal{E}^{\nu\rho\sigma\tau} , \quad (2)$$

where ζ is a coordinate measuring othogonal distance from the 4-dimensional worldsheet of the 3-brane. Here $\mathcal{E}^{\nu\rho\sigma\tau}$ is the antisymmetric tangent element of the worldsheet normalised as $\mathcal{E}^{\nu\rho\sigma\tau} \mathcal{E}_{\nu\rho\sigma\tau} = -4!$. It can be used, in conjunction with the measure tensor $\varepsilon_{\mu\nu\rho\sigma\tau}$ of the 5-dimensional background tensor, to construct the unit normal covector $\lambda_\mu \equiv \nabla_\mu \zeta = (4!)^{-1} \varepsilon_{\mu\nu\rho\sigma\tau} \mathcal{E}^{\nu\rho\sigma\tau}$. The scalar coefficient $e_{\{4\}}$ must necessarily be uniform over the worldsheet in order for the source current to satisfy the conservation law $\nabla_\mu \mathbf{j}^{\mu\nu\rho\sigma} = 0$ that is necessary for gauge invariance. The corresponding “physical” (i.e. gauge invariant) 5-form is defined by the exterior derivative of $A_{\nu\rho\sigma\tau}$ as

$$F_{\mu\nu\rho\sigma\tau} \equiv 5\nabla_{[\mu} A_{\nu\rho\sigma\tau]} = F \varepsilon_{\mu\nu\rho\sigma\tau} , \quad (3)$$

where F is a pseudo scalar (it is not strictly a scalar because its sign is parity dependent) in terms of which the purely external Lagrangian action governing the gauge field has the form

$$\mathcal{L}_{\text{ex}} = \frac{1}{2\alpha} F^2 \quad (4)$$

where α is a coupling constant. Varying this Lagrangian (4) with respect to the 5-dimensional spacetime metric $g_{\mu\nu}$ defines the external stress energy density tensor ($\mathcal{T}_{\text{ex}}^{\mu\nu} = 2\delta\mathcal{L}_{\text{ex}}/\delta g_{\mu\nu} + \mathcal{L}_{\text{ex}} g^{\mu\nu}$) as

$$\mathcal{T}_{\text{ex}}^\mu{}_\nu = -\frac{1}{2\alpha} F^2 g^\mu{}_\nu . \quad (5)$$

A coupling Lagrangian \mathcal{L}_{co} of the most general kind would give rise to another contribution, \mathcal{T}_{co} , to the stress energy density tensor but this can be shown [11] to vanishing in the minimally coupled case (1) we are considering here. Varying the coupling and external Lagrangians with respect to the “brawn” field $A_{\nu\rho\sigma\tau}$ leads to the field equation of motion

$$\nabla^\mu F_{\mu\nu\rho\sigma\tau} = -\alpha \mathbf{j}_{\nu\rho\sigma\tau} . \quad (6)$$

It can thereby be concluded that in the source free region outside the brane the pseudo scalar field F is uniform, i.e. that $\nabla_\mu F = 0$.

An important consequence of this is that outside (but not in) the world brane, the “brawn” stress energy tensor (5) acts just as an effective addition to the cosmological constant Λ that appears in the 5-dimensional Einstein equation according to which the effect of the total stress energy tensor $\mathcal{T}_{\mu\nu}$ (including both external and brane contributions) is given by

$$\mathcal{G}_{\mu\nu} = \kappa \mathcal{T}_{\mu\nu} - \Lambda g_{\mu\nu} , \quad (7)$$

in which $\mathcal{G}_{\mu\nu} \equiv \mathcal{R}_{\mu\nu} - \frac{1}{2} \mathcal{R} g_{\mu\nu}$, where $\mathcal{R}_{\mu\nu}$ is the Ricci tensor and \mathcal{R} is its trace. The traditional unrationalised coupling constant κ is related to the quantity $\kappa_{(5)}$ used by several authors (e.g. [2]), and to the 5-dimensional analogue G of Newton’s constant and its associated gravitational mass scale m_G as defined according to the appropriate [12] rationalisation procedure, by

$$\kappa = \kappa_{(5)}^2 = 6\pi^2 G , \quad G = m_G^{-3} . \quad (8)$$

Thus, in the external region on either side of the brane, substitution of (5) for $\mathcal{T}_{\mu\nu}$ reduces the Einstein equation (7) to the pseudo vacuum form

$$\mathcal{G}_{\mu\nu} = -\Lambda^\pm g_{\mu\nu} , \quad \Lambda^\pm = \Lambda + \frac{\kappa}{2\alpha} F^{\pm 2} , \quad (9)$$

where F^+ and F^- are the constant values taken by the pseudoscalar “brawn” field in the respective external coordinate ranges $\zeta > 0$ and $\zeta < 0$. This minimal coupling case may be seen as the five dimensional analogous of the standard $p = 1$ dimensional case of a charge particle in electromagnetism in which $e_{\{1\}}$ is the particle charge, and $F_{\mu\nu\rho\sigma\tau}$ is just the analog of the electromagnetic tensor $F_{\mu\nu}$ while (6) is the Maxwell equation.

2 The generalised (Birkhoff type) bulk staticity theorem

So long as we are concerned just with cosmological models of homogeneous isotropic type, the consideration that the external gravitational field is of pseudo vacuum type allows to invoke the generalised Birkhoff theorem (see the treatise of Misner *et al.* [13], whose sign conventions we are following). Its original version showed that spherical symmetry necessarily entails the staticity property that was merely assumed as a simplifying ansatz when Schwarzschild first derived his famous solution. A recent demonstration with the range of applicability needed in the present context has recently been provided by Bowcock, Charmousis and Gregory [9], who deal with the case of a 5-dimensional vacuum metric $ds_V^2 = g_{\mu\nu} dx^\mu dx^\nu$ that has the symmetry of a 3-sphere, a 3-plane, or an anti 3-sphere with metric given, respectively for positive, vanishing, or negative curvature parameter k , by

$$ds_{\text{III}}^2 = \frac{d\chi^2}{1 - k\chi^2} + \chi^2 (d\theta^2 + \sin^2\theta d\phi^2) . \quad (10)$$

The conclusion is that wherever equations of the vacuum form (9) are satisfied, and thus on either side of the (minimally coupled) 3-brane under consideration, the metric must be static and hence of the generalised Schwarzschild (anti)-de Sitter form

$$ds_V^2 = r^2 ds_{\mathbb{M}}^2 + \frac{dr^2}{\mathcal{V}} - \mathcal{V} dt^2, \quad (11)$$

with

$$\mathcal{V} = k - \frac{\Lambda^\pm}{6} r^2 - \frac{2G\mathcal{M}^\pm}{r^2}, \quad (12)$$

where \mathcal{M}^+ and \mathcal{M}^- are constants of integration, having the dimensions of mass, that characterise the solution in the respective ranges $\zeta > 0$ and $\zeta < 0$. (This generalisation is analogous to that for the charged black hole case, for which the Birkhoff theorem still applies, giving the static Reissner–Nordström solution.) In the compact case $k > 0$ the quantity r/\sqrt{k} is interpretable as a Schwarzschild type radial coordinate and the mass constants is given by $\mathcal{M}^\pm = k^2 M^\pm$ where M^+ and M^- are interpretable as the masses of distant source distributions, such as other 3-branes (or in their absence, of black holes) on either side. For this positive curvature case it would commonly be convenient to fix the scale so as to obtain $k = 1$, but such a canonical normalisation is not possible in the spacially flat case ($k = 0$) that is accurate within observational uncertainties for the large scale representation of our actual universe.

The fact that the static nature of the external metric is unaffected by the dynamical evolution of the brane means that there is no way it can signal its presence to, and thereby perhaps avoids collision with, any other brane that may be present outside (at a distance whose value as a function of time is what is commonly referred to as the “radion”). The avoidance of this so called “radion instability” problem is one of the motivations for the development of generalisations [14] involving the presence in the “bulk” of a dilatonic scalar field that can carry information from one brane to a neighbouring brane. What we have shown here is that minimal coupling to what is effectively a pseudo scalar field does not offer any such escape from this “radion instability” problem. If only a single brane is present the problem of the “radion” as such will not arise, but if either \mathcal{M}^+ or \mathcal{M}^- is non zero (which, as we shall see, is necessarily the case in the kind of reflection symmetry breaking scenarios envisaged in [3, 4, 5, 6]) one might still need to worry about the possibility of colliding with its source or (if it is source free) of falling into the corresponding black hole.

When one is mainly interested in what is going on in the brane itself rather than in the “bulk” outside, it is commonly convenient to use a brane based reference system depending on the use of the coordinate ζ that measures the orthogonal distance from the worldsheet. In the homogeneous isotropic case this gives an expression of the well known form

$$ds_V^2 = r^2 ds_{\mathbb{M}}^2 + d\zeta^2 - \nu^2 d\tau^2, \quad (13)$$

in which the quantities r and ν are functions just of ζ and τ . The latter is normalised in such a way that in the limit $\zeta \rightarrow 0$ (i.e. on the worldsheet) we have $\nu \rightarrow 1$. It means that τ is interpretable as a measure of the proper cosmological time on the world brane. The corresponding limit $r \rightarrow a$ defines a function $a(\tau)$ interpretable as a cosmologically comoving scale factor, whose proper time derivative \dot{a} specifies the relevant Hubble expansion rate \dot{a}/a .

Although the two versions (11) and (13) have long been separately well known, the relationship between them was not satisfactorily clarified until the recent work by Mukohyama, Shiromizu, and Maeda [15]. These authors showed explicitly how a metric of the static form

(11) can be transformed to the brane based version obtained (in terms of hyperbolic functions of ζ) by Binétruy, Deffayet, Ellwanger and Langlois [10]. The self contained treatment provided here avoids getting into these explicit details [10, 15] by following a more economical approach that does not depend on the particular (explicitly integrable) form (12) of the function \mathcal{V} , and that could thus be useful in a broader context (such as might arise if, as well as the “brawn” field $A_{\mu\nu\rho\sigma}$ considered here, a lower order gauge field, $A_{\mu\nu\rho}$ or $A_{\mu\nu}$ was also present in the bulk).

The essential point is that (in each radial $\{r, t\}$ 2-surface) the distance function ζ is orthogonal to lines of constant τ that are automatically geodesic. The stationarity of the metric (11) therefore implies that the unit tangent vector $x^{\mu'}$ (employing the usual convention that dash and dot indicate partial differentiation with respect to ζ and τ respectively) will contract with the time symmetry Killing vector, as specified by $k_\mu dx^\mu = -d\tau$, to give a constant of the geodesic motion $E = -k_\mu x^{\mu'}$ (that would be interpretable as energy if instead of being spacelike the geodesic was timelike). Though independent of ζ , this quantity E depends in general on the new time variable τ whose normalisation it is now convenient to fix with respect to the rate of change of the scale length a on the worldsheet by imposing the identification $E = \dot{a}$. The definition of E then provides the relation $\dot{t} = \dot{a}/\mathcal{V}$, and hence the unit normalisation condition $g_{\mu\nu} x^{\mu'} x^{\nu'} = 1$ takes the form

$$\dot{a}^2 = r'^2 - \mathcal{V}. \quad (14)$$

In order to preserve coordinate orthogonality, $\dot{x}^\mu x'_\mu = 0$, the required transformation, $dt = \dot{t} d\tau + t' d\zeta$ and $dr = \dot{r} d\tau + r' d\zeta$, must evidently satisfy the condition $\dot{r}r' = \mathcal{V}^2 \dot{t}t'$, so substitution in (11) now leads directly to *q.e.d.*, namely the brane based form (13), in which the coefficient ν can be seen to be given simply by

$$\nu^2 = \dot{r}^2 / \dot{a}^2. \quad (15)$$

This short cut derivation of (13) has made no use of any specific prescription for the dependence of \mathcal{V} on r , but it is now to be observed that when \mathcal{V} is given in terms of the relevant generalised Schwarzschild AdS formula (12), the relations (14) and (15) take precisely the form to which the (bulk) Einstein equations were shown to be reducible by Binétruy *et al.* [10].

The condition of metric continuity on the brane worldsheet (where $r \rightarrow a$) is trivially satisfied by (15), but (14) provides interesting matching conditions. We use square and angle brackets respectively for the difference and average of the values limits on opposite sides, so that in particular for the effective cosmological constant we have

$$[\Lambda] \equiv \Lambda^+ - \Lambda^- = \frac{\kappa}{\alpha} \langle F \rangle [F], \quad \langle \Lambda \rangle \equiv \frac{1}{2} (\Lambda^+ + \Lambda^-) = \Lambda + \frac{\kappa}{2\alpha} \langle F^2 \rangle. \quad (16)$$

One finds that subtraction of (14) gives the relation

$$\langle r' \rangle [r'] = -a^2 \frac{[\Lambda]}{12} - \frac{G[\mathcal{M}]}{a^2}, \quad (17)$$

which is of no interest in the Z_2 reflection symmetric case where all the terms vanish, while averaging (14) gives the more generally useful expansion rate formula

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{\langle \Lambda \rangle}{6} + \frac{\langle r'^2 \rangle - k}{a^2} + \frac{2G\langle \mathcal{M} \rangle}{a^4}. \quad (18)$$

3 Solution of the Junction conditions

To determine the evolution of the system we are interested in, we need to use the well known [13] Darmois Israel junction conditions according to which the active gravitational effect of the surface stress energy density $\bar{T}^{\mu\nu}$ of the brane is governed by the equation

$$[K_{\mu\nu}] - [K]\gamma_{\mu\nu} = \kappa \bar{T}_{\mu\nu} \quad (19)$$

where $\gamma_{\mu\nu}$ and $K_{\mu\nu}$ are respectively the first and the second fundamental forms (K being its the trace), defined as

$$\gamma_{\mu\nu} \equiv g_{\mu\nu} - \lambda_\mu \lambda_\nu, \quad K_{\mu\nu} \equiv -\gamma_\nu^\rho \nabla_\mu \lambda_\rho, \quad (20)$$

$\lambda_\mu = \nabla_\mu \zeta$ being the unit (i.e. $\lambda_\rho \lambda^\rho = 1$) vector normal to the brane (i.e. $\gamma_{\mu\nu} \lambda^\mu = 0$).

It is also necessary to solve the equations governing the passive evolution for the worldsheet, which has often been worked out *ad hoc* in particular applications, but which can also be expressed [11] in a simple generally valid form as

$$\bar{T}^{\mu\nu} \langle K_{\mu\nu} \rangle = \bar{f}, \quad (21)$$

where \bar{f} is the force density acting on the brane. This will be given in terms of the contraction of the external (bulk) stress energy density as

$$\bar{f} = -\lambda_\mu \lambda_\nu [T_{\text{ex}}^{\mu\nu}]. \quad (22)$$

This must of course vanish in the reflection symmetric configurations that are most commonly considered, but need not do so in the more general context considered here. It can be seen from (6) and from the expression (5) for $F_{\mu\nu\rho\sigma\tau}$ that $\lambda_\mu \nabla^\mu F = \alpha e_{\{4\}} \delta[\zeta]$, which implies that

$$[F] = \alpha e_{\{4\}}. \quad (23)$$

The force density is found to have a constant but generically non vanishing value given by

$$\bar{f} = e_{\{4\}} \langle F \rangle = \kappa^{-1} [\Lambda]. \quad (24)$$

In order to apply the preceeding formulae one has to work out the exact expressions of the first and second fundamental forms as defined in equation (20). The first fundamental form is obviously given by

$$ds_{\text{IV}}^2 = \gamma_{\mu\nu} dx^\mu dx^\nu = a^2 ds_{\text{III}}^2 - d\tau^2, \quad (25)$$

and the second fundamental form by

$$K_{\mu\nu} dx^\mu dx^\nu = -ar' ds_{\text{III}}^2 + \nu' d\tau^2. \quad (26)$$

We also need to know the form of the worldsheet stress energy density tensor $\bar{T}^{\mu\nu}$. To be compatible with the hypothesis of a homogeneous isotropic geometry, $\bar{T}^{\mu\nu}$ must itself be of the homogeneous isotropic form, i.e.

$$\bar{T}^{\mu\nu} = \mathcal{U} u^\mu u^\nu - \mathcal{T} (\gamma^{\mu\nu} + u^\mu u^\nu) \quad (27)$$

with respect to the preferred unit vector defined by $u_\mu = -\nabla_\mu \tau$. Here \mathcal{U} is the total energy density and \mathcal{T} is the total brane tension. It is normally assumed that the directly observable

energy density ρ and pressure P (i.e. of the cosmic fluid) represent small deviations from an isotropic (inflationary) limit state given by $\overline{\mathcal{T}}^{\mu\nu} = -\mathcal{T}_\infty \gamma^{\mu\nu}$ where \mathcal{T}_∞ is a fixed tension (approached by both \mathcal{T} and \mathcal{U} in the limit $a \rightarrow \infty$) in terms of which the actual tension and energy density are given by

$$\mathcal{T} \equiv \mathcal{T}_\infty - P, \quad \mathcal{U} \equiv \mathcal{T}_\infty + \rho. \quad (28)$$

Substitution of (27) in (19) gives explicit jump conditions of the well known [2] form

$$[r'] = -\frac{a\kappa}{3}\mathcal{U}, \quad [\nu'] = \frac{\kappa}{3}(2\mathcal{U} - 3\mathcal{T}). \quad (29)$$

Now, from (21) we obtain the relation

$$\mathcal{U}\langle\nu'\rangle + 3\mathcal{T}\frac{\langle r'\rangle}{a} = \overline{f}, \quad (30)$$

which differs from the usual version by the presence of the force density term on the right.

Using the expression (29) for $[r']$, we obtain, from (17), that $2\langle r'\rangle$ (which is the same as the quantity “ d ” introduced as a measure of the deviation from reflection symmetry by Davis *et al.* [5]) is given by

$$2\langle r'\rangle = \frac{3}{\kappa\mathcal{U}} \left(\frac{a[\Lambda]}{6} + \frac{2G[\mathcal{M}]}{a^3} \right). \quad (31)$$

This generalises the corresponding formula of Davis *et al.* by the inclusion of the $[\Lambda]$ term representing the effect of the “brawn” force, and it also provides a more specific interpretation of their “ F ” term by showing that (as observed by Ida [3]) it is proportional to the (reflection symmetry breaking) difference $[\mathcal{M}]$ between the mass terms on opposite sides, and hence that it would have to be absent if both mass terms were zero (as was assumed in the work of Deruelle and Doležal [6] and Perkins [7]). It is to be observed that the gravitational coupling coefficients can be cancelled out of (31) and hence also out of the expressions for the components of the mean value of the first fundamental form (26), which vanish in the reflection symmetric case, and which will be given in the general case by

$$\mathcal{U}\langle r'\rangle = \frac{[\mathcal{M}]}{2\pi^2 a^3} + \frac{a}{4}e_{\{4\}}\langle F\rangle, \quad \mathcal{U}\langle\nu'\rangle = -\frac{3[\mathcal{M}]\mathcal{T}}{2\pi^2 a^4 \mathcal{U}} + \left(1 - \frac{3\mathcal{T}}{4\mathcal{U}}\right)e_{\{4\}}\langle F\rangle. \quad (32)$$

Using (31) and the first equation of (29) respectively for $\langle r'\rangle$ and $[r']$ and using the decomposition $\langle r'^2\rangle = \langle r'\rangle^2 + [r']^2/4$ to evaluate $\langle r'^2\rangle$ in (18), we can now proceed directly to the relevant generalisation of the Friedmann equation, which takes the form

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{\kappa\mathcal{U}}{6}\right)^2 + \frac{\langle\Lambda\rangle}{6} - \frac{k}{a^2} + \frac{2G\langle\mathcal{M}\rangle}{a^4} + \frac{1}{(2\kappa\mathcal{U})^2} \left(\frac{[\Lambda]}{2} + \frac{6G[\mathcal{M}]}{a^4} \right)^2. \quad (33)$$

This agrees with what was obtained in a rather more circuitous manner by Stoica *et al* [8], and by Bowcock *et al.* [9], who were the first to undertake a systematic investigation of reflection symmetry breaking scenarios with the degree of generality considered here, but who used a system in which the 5-dimensional gravitational coupling constant κ was set to unity, thereby obscuring the way it affects the various terms. As well as directness, the present approach has the advantage of showing more explicitly how the various coupling coefficients contribute, and in particular how κ actually cancels out of the formulae (32) and, as will be demonstrated immediately below, from last term of (33) which is the only one whose presence depends on reflection symmetry breaking.

4 Investigation of the parameter space

In order to interpret the Friedmann-like equation (33), we use (28) to rewrite it in terms of the ratio $\varepsilon \equiv \rho/\mathcal{T}_\infty$, as

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G_4}{3}\rho - \frac{k}{a^2} + \frac{\Lambda_4}{3} + \frac{2G\langle\mathcal{M}\rangle}{a^4} + (\pi^2 G\rho)^2 \\ &+ \frac{(1+\varepsilon)^{-2}}{(2\mathcal{T}_\infty)^2} \left\{ \frac{[\mathcal{M}]}{\pi^2 a^4} \left(\frac{[\mathcal{M}]}{\pi^2 a^4} + e_{\{4\}}\langle F \rangle \right) + \left(\frac{e_{\{4\}}\langle F \rangle}{2} \right)^2 (3+2\varepsilon)\varepsilon^2 \right\} \end{aligned} \quad (34)$$

in which the first three terms of the r.h.s. are those of the standard Friedmann equation, with the effective 4-dimensional Newton constant G_4 and the effective 4-dimensional cosmological constant Λ_4 given by

$$8\pi G_4 = \frac{6}{\mathcal{T}_\infty} \left((\pi^2 G\mathcal{T}_\infty)^2 - \left(\frac{e_{\{4\}}\langle F \rangle}{4\mathcal{T}_\infty} \right)^2 \right), \quad (35)$$

and

$$2\Lambda_4 = \langle\Lambda\rangle + 6 \left((\pi^2 G\mathcal{T}_\infty)^2 + \left(\frac{e_{\{4\}}\langle F \rangle}{4\mathcal{T}_\infty} \right)^2 \right). \quad (36)$$

In terms of the brane mass scale given by $m_\infty^4 = \mathcal{T}_\infty$, and of the ordinary Planck mass scale given by $m_P^{-2} \equiv G_4$, it is convenient to parametrise the relative importance in (35–36) of the gauge field contribution $\langle F \rangle$ and of the bulk gravitational coupling constant given by $m_G^{-3} \equiv G$, in terms of a dimensionless hyperbolic angle χ such that

$$e_{\{4\}}\langle F \rangle = \left(\frac{\pi}{3}\right)^{1/2} \frac{8m_\infty^6 \text{sh } \chi}{m_P}, \quad G = \left(\frac{\pi}{3}\right)^{1/2} \frac{2 \text{ch } \chi}{\pi^2 m_\infty^2 m_P}. \quad (37)$$

Besides the three independent parameters involved in (37), namely χ , m_∞ and m_P , of which only the latter is experimentally known in advance, this model involves four more independent parameters which are the bulk mass scales \mathcal{M}^\pm , the ordinary cosmological constant Λ_4 and the cosmological curvature scale k . Only the two latter are roughly known in advance, neither of them differing from zero by an amount that can be reliably measured yet. Since Λ_4 is at most of the order of the present day cosmological closure value (i.e. $m_{\Lambda_4} \lesssim 10^{-60} m_P$), it is effectively negligible for our present purpose, i.e. we can set $\Lambda_4 = 0$. This means that the effective bulk cosmological constants and the corresponding curvature length and mass scales $\ell_\Lambda = m_\Lambda^{-1}$ as defined by $-\Lambda \equiv 6\ell_\Lambda^{-2}$ on either side of the brane will be given (using (37) for $\langle\Lambda\rangle$, and using (24) and (37) for $[\Lambda]$) by

$$-\Lambda^\pm = 6m_\Lambda^{\pm 2} = 8\pi \frac{m_\infty^4}{m_P^2} e^{\mp 2\chi}, \quad \ell_\Lambda^\pm = \left(\frac{3}{\pi}\right)^{1/2} \frac{m_P}{2m_\infty^2} e^{\pm\chi}. \quad (38)$$

In order to place some limits on the four independent unknown constants \mathcal{T}_∞ , χ , and the bulk mass constants it is convenient to replace \mathcal{M}^\pm by variable mass parameters

$$\mu^\pm \equiv 2G\mathcal{M}^\pm \left(1 \pm \frac{\text{th } \chi}{(1+\varepsilon)^2} \right). \quad (39)$$

With this notation, we can convert (34) to the form

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\rho}{3m_{\text{p}}^2} - \frac{k}{a^2} + \frac{\Lambda_4}{3} + \Delta, \quad (40)$$

in which the final term Δ represents the deviation from the traditional Friedmann equation and is expressible as

$$\Delta = (\pi^2 G \rho)^2 \left(1 + \frac{(3 + 2\varepsilon) \text{th}^2 \chi}{(1 + \varepsilon)^2}\right) + \left(\frac{[\mathcal{M}]}{2\pi^2 a^4 \mathcal{T}_\infty (1 + \varepsilon)}\right)^2 + \frac{\langle \mu \rangle}{a^4}, \quad (41)$$

in which the first two terms are manifestly positive definite. Furthermore if (unlike some authors who, in scenarios involving more than one brane, have envisaged negative mass densities) we make the traditional assumption that all external mass distributions must be positive (hence $\mathcal{M}^\pm \geq 0$), then it can be deduced from the expression (39) that both variables μ^+ and μ^- must also be everywhere positive, though in the case of the latter only marginally if χ is large.

Before proceeding to exploit these positivity properties for the purpose of placing limits on the parameters involved, we need to start by choosing a convenient normalisation for the scale factor a . Since the spacial curvature factor k is too small to have been reliably measured yet ($k = 0$ being a plausible possibility and certainly a good approximation), the mathematically convenient choice $k = 1$ is in practice unavailable. Whatever k may be, a physically convenient choice is to normalise the present value of the scale factor a to agree with the length scale characterising the cosmological background radiation temperature Θ , i.e. to simply to take $a = \Theta^{-1}$. This normalisation has the advantage that it holds when extrapolated backwards in time so long as photon creation processes remain insignificant, which means ever since the temperature dropped below the electron positron pair creation threshold given by $\Theta \approx 2m_e$.

In particular the temperature Θ_{N} of nucleosynthesis, which is of greatest interest for the purpose of obtaining rigorous observational constraints, lies in the range $0 < \Theta < 2m_e$ in which the product $a\Theta$ remains constant. Moreover throughout the radiation dominated era during which

$$\rho = \frac{\pi^2 g_*}{30} \Theta^4 \quad (42)$$

the order of magnitude relation $\Theta \approx a^{-1}$ will still be valid as a useful approximation so long as the dimensionless coefficient g_* representing the effective number of relativistic degrees of freedom remains comparable with its usual value $g_* \simeq 10.75$ at nucleosynthesis. Under such conditions, with k and Λ_4 now set to zero (since they are negligible in that era), one can rewrite (40) as

$$\left(\frac{\dot{\Theta}}{\Theta}\right)^2 = \frac{8\pi\rho}{3m_{\text{p}}^2} + \Delta. \quad (43)$$

The deviation term (41) can be expressed in terms of correspondingly normalised “bulk” mass scales $m_{\text{B}}^\pm \equiv \mathcal{M}^\pm / 2\pi^2$ (representing the distant external mass associated with a small comoving volume of size a^3) as

$$\Delta = \frac{4\pi}{3\mathcal{T}_\infty} \left(\frac{\rho \text{ch} \chi}{m_{\text{p}}}\right)^2 \left(1 + \frac{(3 + 2\varepsilon) \text{th}^2 \chi}{(1 + \varepsilon)^2}\right) + \left(\frac{[m_{\text{B}}]\Theta^4}{\mathcal{T}_\infty (1 + \varepsilon)}\right)^2 + \frac{\mu^+ \Theta^4}{2} + \frac{\mu^- \Theta^4}{2}. \quad (44)$$

Since each term is positive, the condition that Δ should be small compared with the leading term in (43) when $\Theta \approx \Theta_N$ imposes this smallness requirement on each of the four separate terms in (44). The constraint on the first term gives the basic requirement that

$$\Theta_N^2 \text{ch } \chi \ll \sqrt{\frac{15}{2\pi^2 g_*}} m_\infty^2, \quad (45)$$

with the convenient corollary that the value of ε at the time of nucleosynthesis satisfies $\varepsilon_N \ll 1$ throughout the range $\Theta \leq \Theta_N$ (and considerably beyond if χ is large). From the second term, one obtains the requirement

$$|[m_B]| \ll \sqrt{\frac{4\pi^3 g_*}{45}} \frac{m_\infty^4}{m_P \Theta_N^2}. \quad (46)$$

One can understand these two constraints by expanding Δ in series with respect to $\varepsilon \ll 1$; they simply state that the terms scaling faster than Θ^4 (e.g. $\Theta^8 \dots$) are negligible at the time of nucleosynthesis, thus explaining the dependence on Θ_N in (45) and (46). The third and fourth terms in (44) scale as radiation, thus behaving like extra relativistic degrees of freedom which are constrained by the fact that g_* cannot deviate for more than 20% from its expected value $g_* = 10.75$. If we choose an orientation convention so that $e_{\{4\}} \langle F \rangle$ is positive (thus implying that $\chi \geq 0$) then the constraint on the third term of (44) gives the requirement

$$m_B^+ \text{ch } \chi \ll \frac{2\pi^2 g_*}{15} \sqrt{\frac{\pi}{3}} \frac{m_\infty^2}{m_P}. \quad (47)$$

Due to the possibility of partial cancellation, the condition obtained from the last term in (44) is less restrictive: by (45), since $\varepsilon_N \text{ch}^2 \chi \ll 1$, it is expressible as

$$\frac{m_B^-}{\text{ch } \chi} \ll \frac{2\pi^2 g_*}{15} \sqrt{\frac{\pi}{3}} \frac{m_\infty^2}{m_P}. \quad (48)$$

As expected, these two conditions are independent of Θ_N since the contributions of the terms in μ^\pm are proportional to that of the radiation.

Using that $|[m_B]| < m_B^+ + m_B^-$ and that $\text{ch } \chi + 1/\text{ch } \chi < 2\text{ch } \chi$, we deduce that $|[m_B]| < 4\pi^2 \sqrt{\pi/3} m_\infty^2 / (15m_P)$, and thus that the condition (45) on χ leads to the constraint (46). This means that (46) is not needed as an independent condition, but is an automatic consequence of the other three conditions (45), (47) and (48).

Furthermore experimental limits on deviations from the Newtonian law of gravity imply an upper bound [17] on the length scale ℓ_Λ , and a corresponding lower bound on the mass scale m_Λ in the bulk. Since no such deviation has been detected above a millimeter [16], the present bound is $m_\Lambda \gtrsim 10^{-3} \text{eV}$, which happens to be comparable with present temperature Θ_0 of the cosmic microwave background, and can thus be written as

$$m_\Lambda \gtrsim \Theta_0. \quad (49)$$

Previous discussions [17] of this bound considered only scenarios characterised by Z_2 reflection symmetry. When this symmetry is broken, the bound on the cosmological length scale, and by (29 and (32) on the corresponding asymptotic curvature limit $K_{\mu\nu}^\pm \sim \pm m_\Lambda^\pm \gamma_{\mu\nu}$ as $a \rightarrow \infty$, must presumably be satisfied on each side separately. Since our orientation convention is such that ℓ_Λ^+ is greater than ℓ_Λ^- we deduce that in the generic case the mass values given by (38) will be characterised by

$$m_\Lambda^- \geq m_\Lambda^+ \gtrsim \Theta_0. \quad (50)$$

5 Implications and conclusions

The requirement (50) is evidently most severe for what, according to our convention, is the negative side of the brane, so by (38) it gives an order of magnitude restriction expressible as

$$m_\infty^2 \gtrsim \sqrt{\frac{3}{\pi}} \Theta_0 m_P \operatorname{ch} \chi. \quad (51)$$

This is evidently strong enough to ensure that (45) will be satisfied as an automatic consequence. Nucleosynthesis and laboratory experiments on the gravitational force thus provide just three independent constraints. One of them is given by (51), and the two others are (47) and (48), which can be combined as

$$m_B^\pm \ll \frac{2\pi^2}{15} \sqrt{\frac{\pi}{3}} \frac{m_\infty^2}{m_P} (\operatorname{ch} \chi)^{\mp 1}. \quad (52)$$

If m_∞ has the minimum value allowed by (51), i.e. $m_\infty^2 \approx (3/\pi)^{1/2} \Theta_0 m_P \operatorname{ch} \chi$, then (52) simply gives

$$m_B^+ \ll \frac{2\pi^2}{15} \Theta_0, \quad m_B^- \ll \frac{2\pi^2}{15} \Theta_0 (\operatorname{ch} \chi)^2. \quad (53)$$

To estimate the orders of magnitude imposed by the three constraints (51) and (52), we use the approximate values $\Theta_0 \sim 2 \times 10^{-13} \text{ GeV}$, $m_P \sim 10^{19} \text{ GeV}$ and $\Theta_N \sim 1 \text{ MeV}$, respectively for the cosmic background radiation temperature, the Planck mass and the nucleosynthesis temperature. It is convenient to distinguish two regimes as follows.

1. Type I scenarios, characterised by $\chi \lesssim 1$: in these scenarios the contribution $e_{\{4\}} \langle F \rangle$ is relatively unimportant both in (35) and (36). Using $\operatorname{ch} \chi \simeq 1$, we deduce from (51) that the mass scale specifying the limiting value of the brane tension, $\mathcal{T}_\infty = m_\infty^4$, must satisfy

$$m_\infty \gtrsim 10^6 \Theta_N \approx 1 \text{ TeV}, \quad (54)$$

which is the same as in the usual Z_2 reflection symmetric scenarios. In this case, from (52), the two external mass scales satisfy the same constraint

$$m_B^\pm \ll \frac{2\pi^2}{15} \sqrt{\frac{\pi}{3}} \frac{m_\infty^2}{m_P}, \quad (55)$$

which in the limit $m_\infty \approx 1 \text{ TeV}$ gives $m_B^\pm \ll 10^{-10} \text{ MeV}$.

2. Type II scenarios, characterised by $\chi \gg 1$: in these scenarios the contribution $e_{\{4\}} \langle F \rangle$ from the gauge field is very large, so there has to be a fine tuning between its value and that of the correspondingly large five dimensional gravitational constant G to as to get the relatively small observed value of G_4 by (35). In this case (54) is to be replaced by a more restrictive condition of the form

$$m_\infty \gtrsim (\operatorname{ch} \chi)^{1/2} \text{ TeV}. \quad (56)$$

In such scenarios the external mass scales satisfy different constraints. In particular if the brane mass scale has the minimum value $m_\infty \approx (\operatorname{ch} \chi)^{1/2} \text{ TeV}$ allowed by (56) we get $m_B^+ \ll 10^{-10} \text{ MeV}$ which is the same as before, but on the other side (52) gives the potentially much less restrictive condition $m_B^- \ll 10^{-10} (\operatorname{ch} \chi)^2 \text{ MeV}$.

To sum up, in this article we have introduced the simplest conceivable coupling to an external gauge field, which has been shown to be compatible with staticity. Like the effect [3, 4, 5] of external masses, this “brawn” coupling breaks the Z_2 reflection symmetry and gives a mechanism for the set up considered for instance by Deruelle and Doležel [6], Stoica *et al.* [8], Perkins [7], Davis *et al.* [5] and Bowcock *et al.* [9]. We have given a simple self contained derivation of the generalised Friedmann equation which now depends on five adjustable parameters, and we have used its Friedmannian limit to identify the four dimensional Newton constant and cosmological constant. The 4 independent parameters involved, namely the hyperbolic angle χ , the brane tension mass scale m_∞ , and the external masses per thermal volume m_B^+ and m_B^- (2 more than are needed for the reflection symmetric case in which $\chi = 0$ and $m_B^+ = m_B^-$) have been shown to be subject to 3 independent constraints (one more than in the symmetric case) of which two arise from nucleosynthesis measurements and one from gravitational laboratory experiments above a millimeter. We distinguish two kinds of regime depending on the value of the hyperbolic angle χ that characterises the relative importance of the “brawn” field. In the type II case for which this parameter is large the external mass scale can (on one but not both sides) be much larger than in the usual reflection symmetric case, but for this to be possible the brane mass scale m_∞ must also be much larger than its usual lower limit of the order of a TeV.

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